Surprising results on task assignment for high-variability workloads

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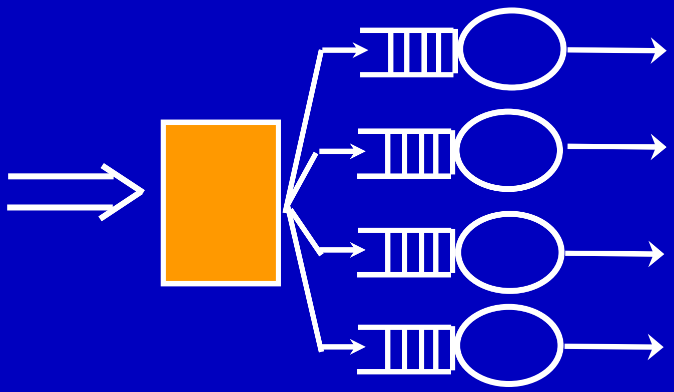
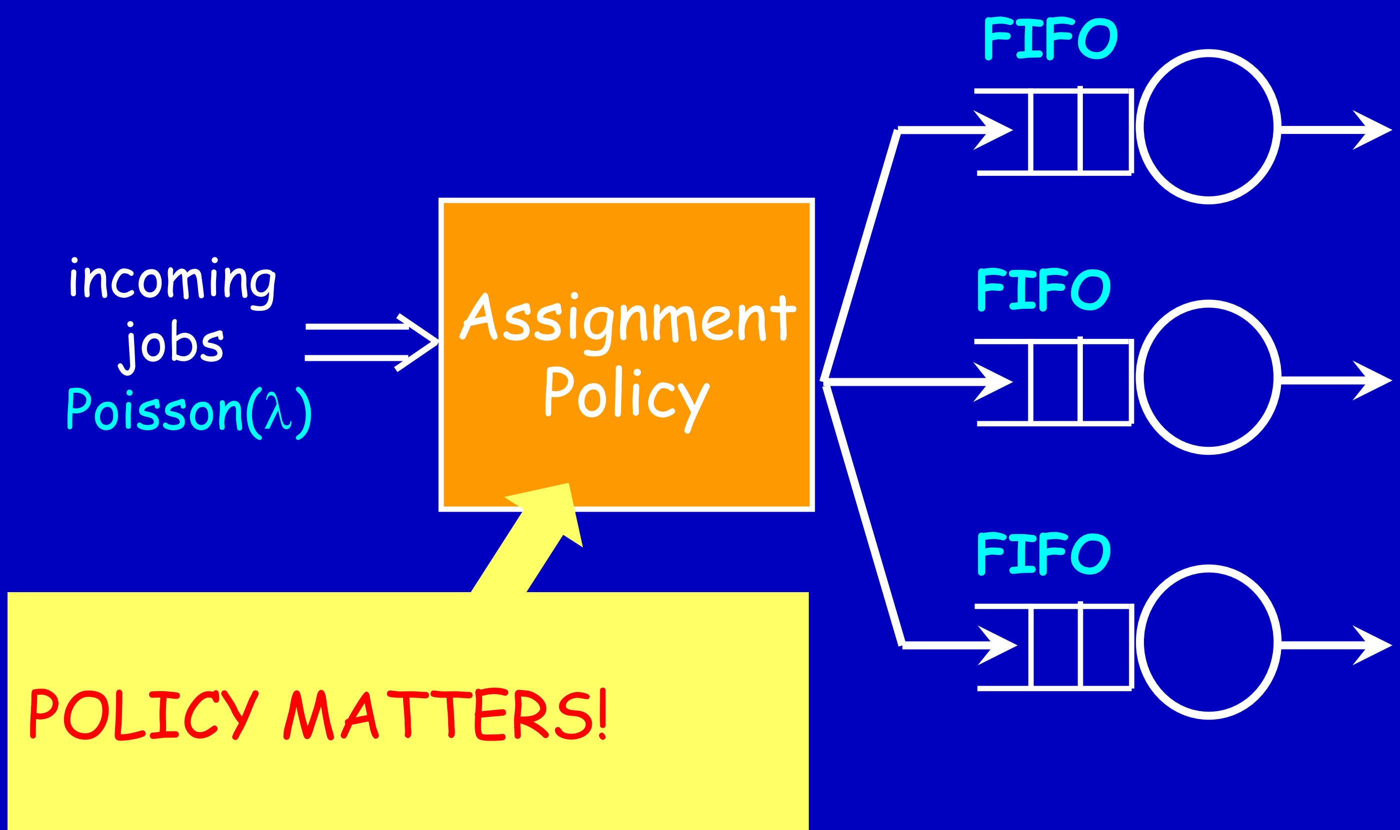
Andrew Young, Morgan Stanley



Server farm model

Goal: Minimize mean response time: E[T]



n servers

general i.i.d. job sizes ~ X

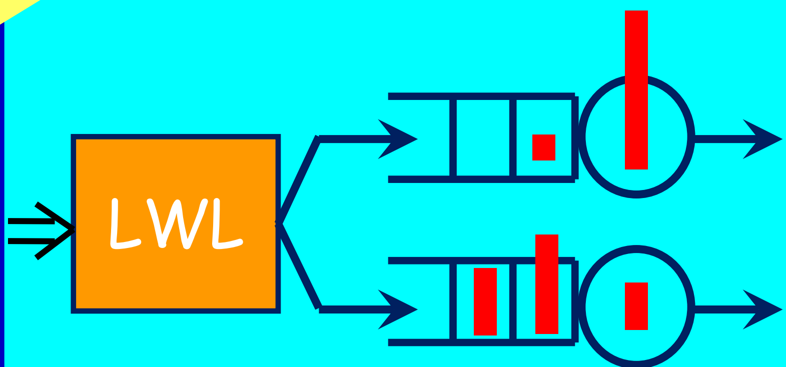
$$C^2 = \frac{\mathrm{var}(X)}{E[X]^2}$$

$$\rho = \lambda E[X] \le n$$

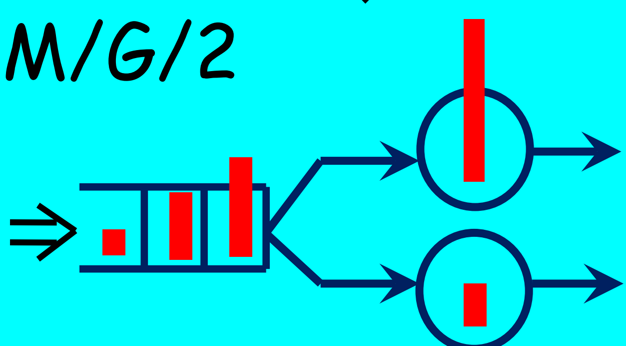
Good Answers

LWL (L… st Work Left)
S… ob to host with
… t remaining work.

+High throughput



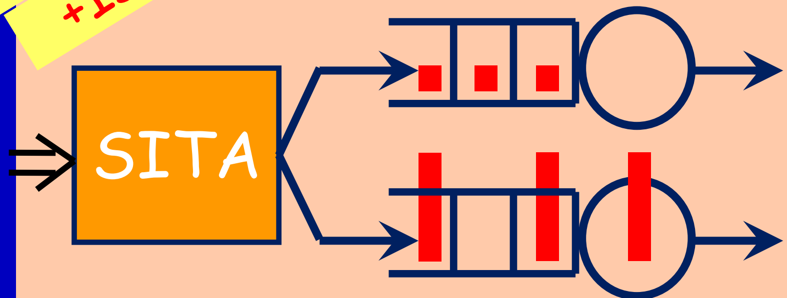
M/G/2



SITA … erval)
Spli… d on size

+Protects against high variability

+Isolation for smalls



Prior Work on SITA

SITA in Practice

- Supercomputing Centers
  [Hotovy, Schneider, O'Donnell 96]
  [Schroeder, Harchol-Balter 00]
- Manufacturing Centers
  [Buzacott, Shanthikumar 93]
- File Server Farms
  [Cardellini, Colajanni, Yu 01]
- Supermarkets

Optimizing SITA cutoffs

- [Harchol-Balter,Crovella,Murta 98]
- [Bachmat, Sarfati 08]
- [Sarfati 08]
- [Harchol-Balter, Vesilo 08]

SITA variants

- [Harchol-Balter 00]
- [Harchol-Balter 02]
- [Thomas 08]
- [Tari,Broberg,Zomaya, Baldoni 05]
- [Fu, Broberg, Tari 03]

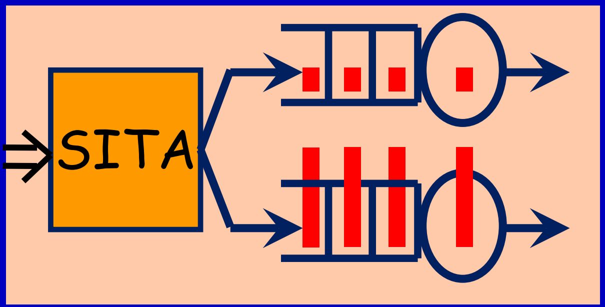
SITA vs. LWL

- [Broberg, Tari, Zeephongsek 0
- [Harchol-Balter 02]
- [Ciardo, Ris
-

vella, Murta 99]
99]
- [Tari, Broberg, Zomaya, Baldoni 05]
- [Thomas 08]

All conclude SITA far superior for high variability

Can't prove anything because it's not true!



Alternative Title:

The TRUTH about SITA, under very high job size variability

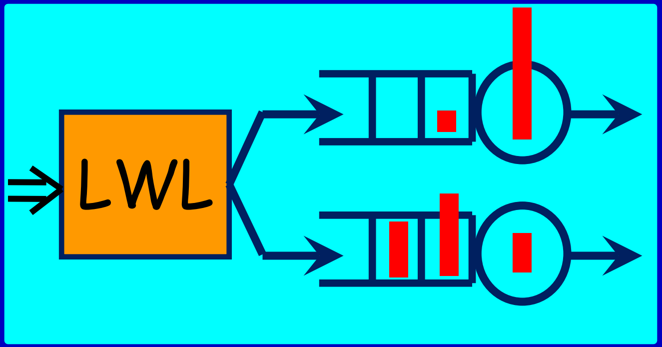
$$C^2 = \frac{\mathrm{var}(X)}{E[X]^2} \to \infty \quad \text{while} \quad E[X]: \textit{fixed}$$

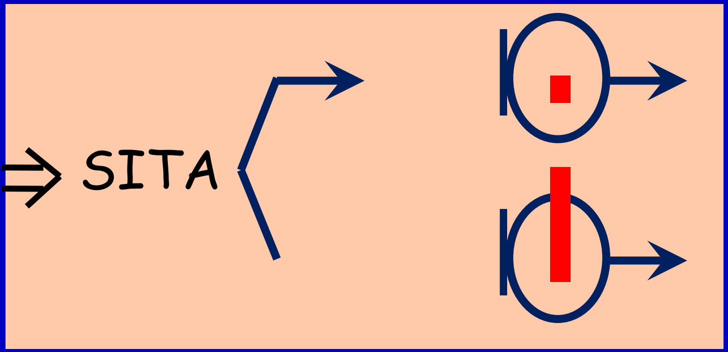
Q: In this talk we will show ...

as $C^2 \to \infty$

a) SITA diverges & LWL diverges?
b) SITA converges & LWL diverges ?
c) SITA diverges & LWL converges?
d) SITA converges & LWL converges?

A: All of the above



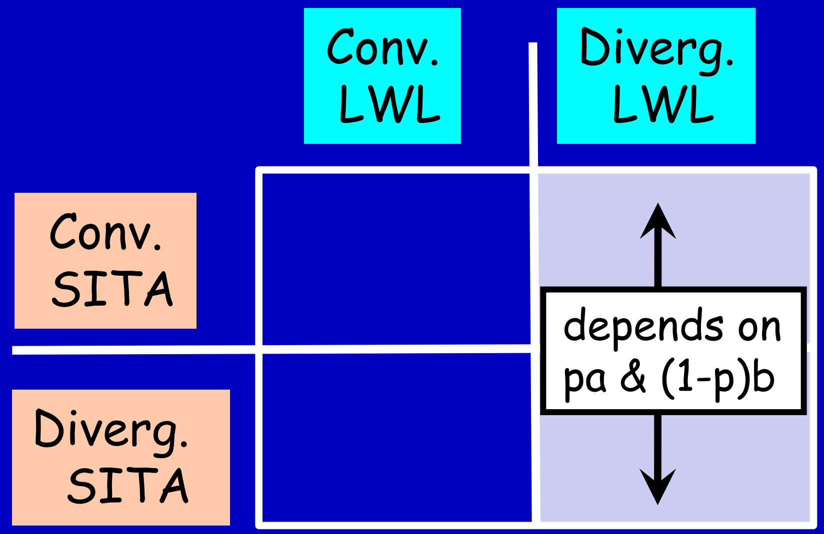


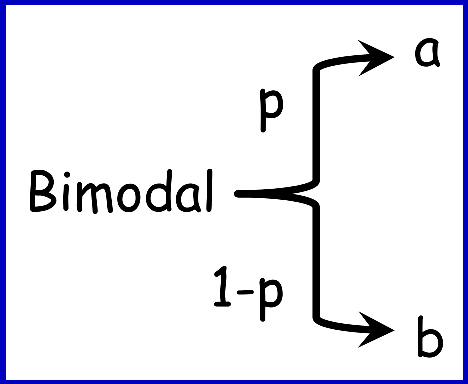
Q: In this talk we will show … as $C^2 \rightarrow \infty$

	Convergent LWL	Divergent LWL
Convergent SITA	✓	✓
Divergent SITA	✓	✓

Looking for simple job size distributions to illustrate each.

Results (2 server system)





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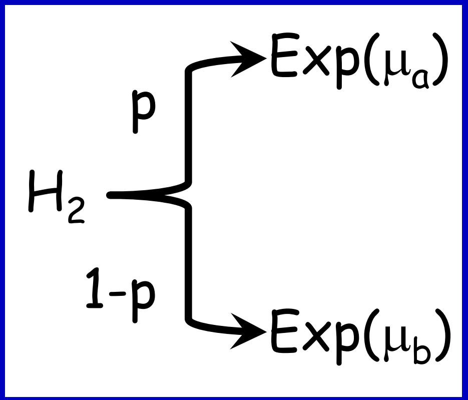
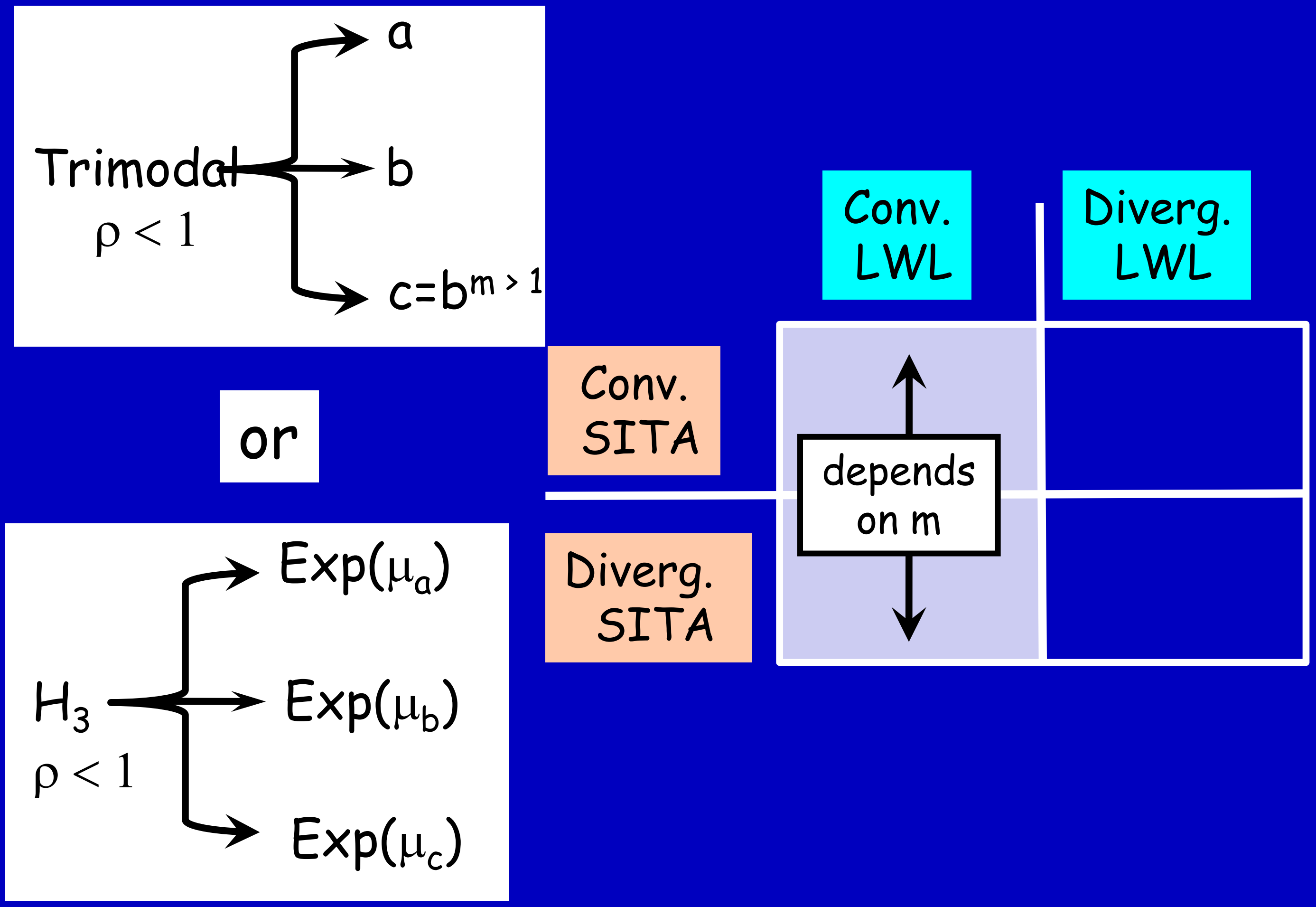
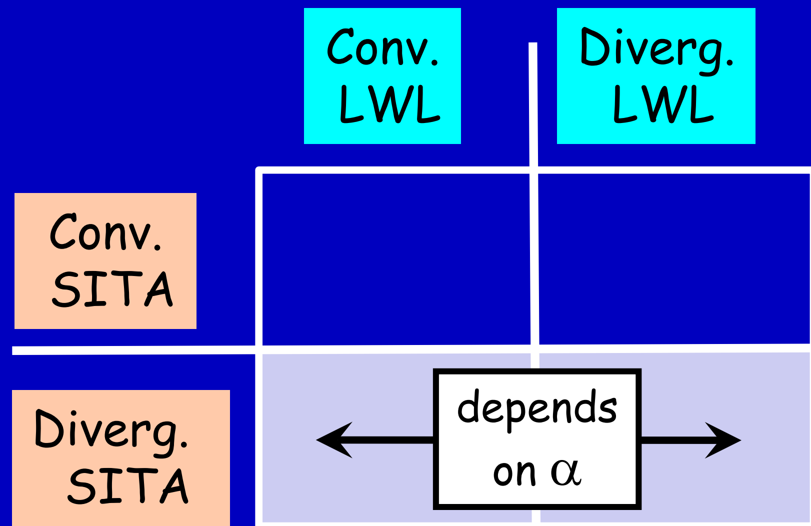
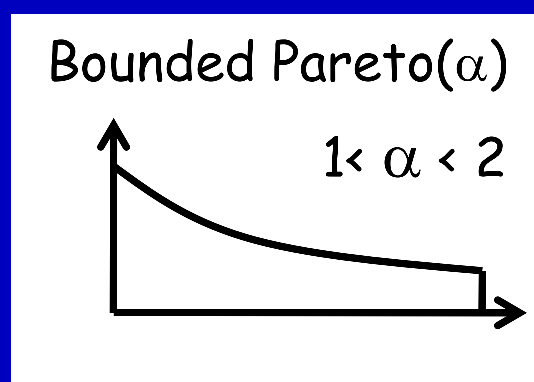


Results (2 server system)



Results (2 server system)





Bimodal Results

	Conv. LWL	Diverg LWL
Conv. SITA		depends ρ_a & ρ_b
Diverg SITA		

$pa = QE[X]$

$$X \sim \begin{cases} a & p \\ b & 1-p \end{cases}$$

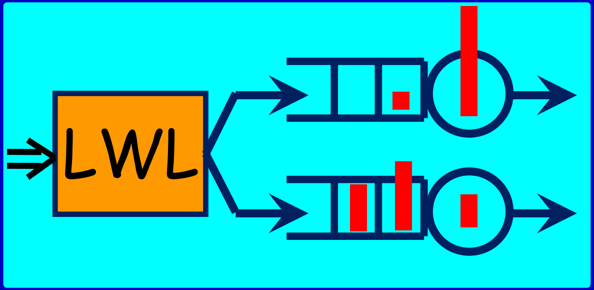
$(1-p)b = (1-Q)E[X]$

Lemma: *As* $C^2 \rightarrow \infty$, but E[X], Q: const,
a's get little smaller → QE[X]
b's get much bigger → ∞
p → 1

THM: If $\rho_a < 1$ & $\rho_b < 1$
➔ Convergent SITA

THM: LWL always diverges.

Understanding LWL



Isn't LWL always bad for high C^2?

It depends ...

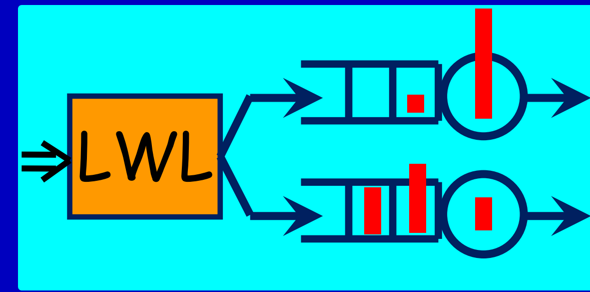
But shorts stuck behind longs, so $E[T] \rightarrow \infty$

Need 2 longs for this to be a problem!

So we need: Pr{ 2 longs } * E[T| 2 longs] ?

Suffices to just look at $E[X^{3/2}]$.

Understanding LWL



Thm: [Scheller-Wolf, Sigman 97], [Scheller-Wolf, Vesilo 06] (2 SERVERS)

$$\text{If } \mathrm{E}[X^{3/2}] < \infty \ \& \ \rho < 1 \Rightarrow \mathrm{E}[T]^{\mathrm{LWL}} < \infty$$

$$(\Leftarrow \text{usually})$$

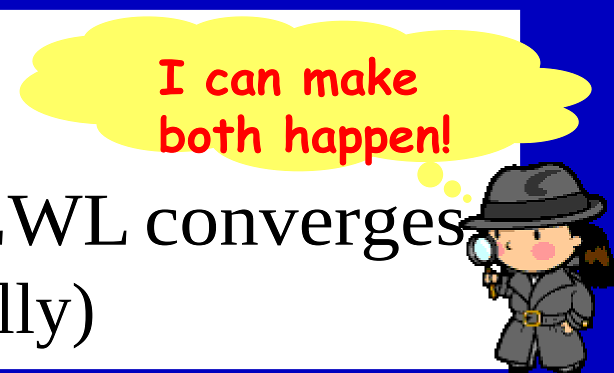
1 spare server

$C^2 \to \infty$

Thm:

$$\text{If } \left\{ \boxed{\begin{array}{c} E[X^{3/2}]: \text{bounded} \\ \text{while } C^2 \to \infty \end{array}} \right\} \& \ \rho < 1 \ \Rightarrow \text{LWL converges}$$

$$(\Leftarrow \text{usually})$$



Bimodal Results

	Conv. LWL	Diverg LWL
Conv. SITA		depends ρ_a & ρ_b
Diverg SITA		

$pa = QE[X]$

$$X \sim \begin{cases} a & p \\ b & 1-p \end{cases}$$

$(1-p)b = (1-Q)E[X]$

Lemma: As $C^2 \to \infty$, but $E[X]$, Q: const,

$a \to QE[X]$, $b \to \infty$, $p \to 1$

THM: LWL always diverges.

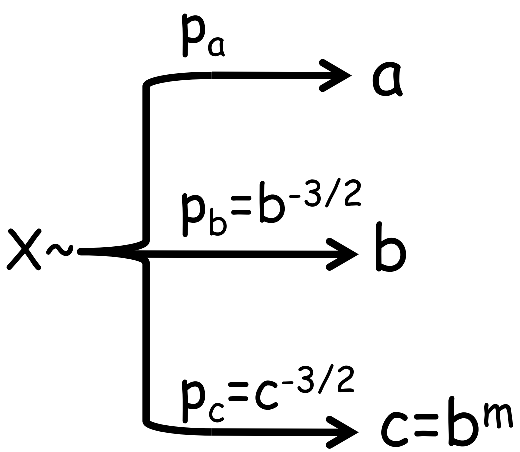
$$E[X^{3/2}] = pa^{3/2} + (1-p)b^{3/2}$$
$$= QE[X]\sqrt{a} + (1-Q)E[X]\sqrt{b}$$
$$\to \infty \ \ (\text{as } C^2 \to \infty)$$

THM: If $\rho_a < 1$ & $\rho_b < 1$

➔ Convergent SITA

Trimodal Results

<table>
<tr><th></th><th>Conv. LWL</th><th>Diverg LWL</th></tr>
<tr><td>Conv. SITA</td><td rowspan="2">depends on m</td><td></td></tr>
<tr><td>Diverg SITA</td><td></td></tr>
</table>



<u>Lemma</u>: *As* $C^2 \to \infty$, but E[X]: const,

$a \to E[X]$

$b \to \infty$, $c \to \infty$

$p_a \to 1$

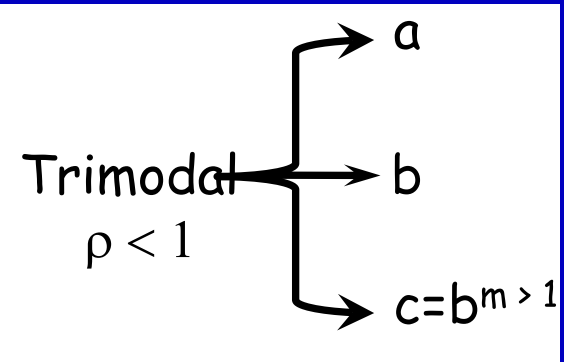
<u>THM</u>: If $m \le 3$, SITA converges
If $m > 3$, SITA diverges

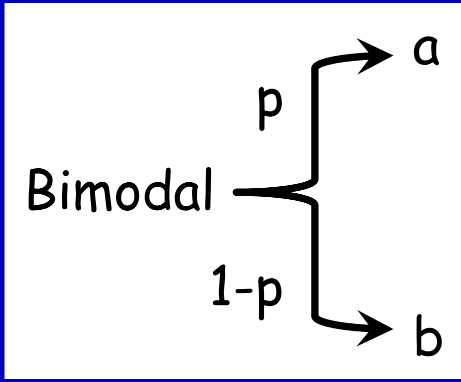
<u>THM</u>: LWL always converges for $\rho < 1$

$$E[X^{3/2}] = p_a a^{3/2} + p_b b^{3/2} + p_c c^{3/2}$$

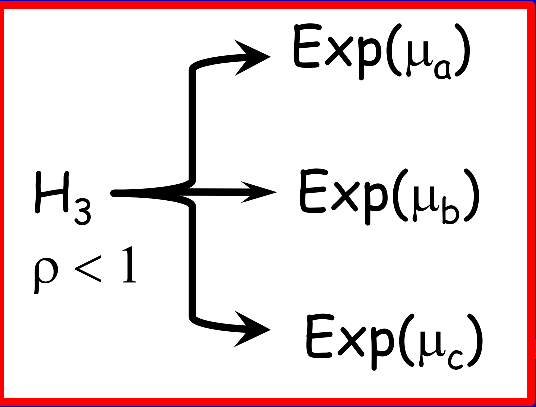
$$\to E[X]^{3/2} + 1 + 1$$

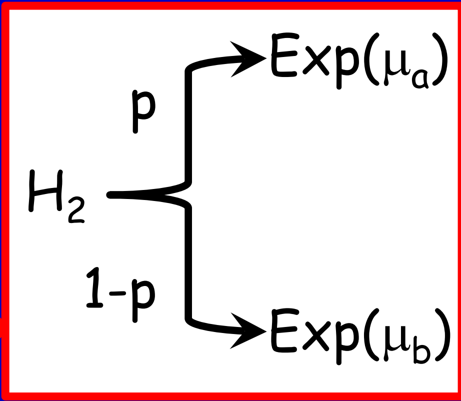
Results
(2 server system)





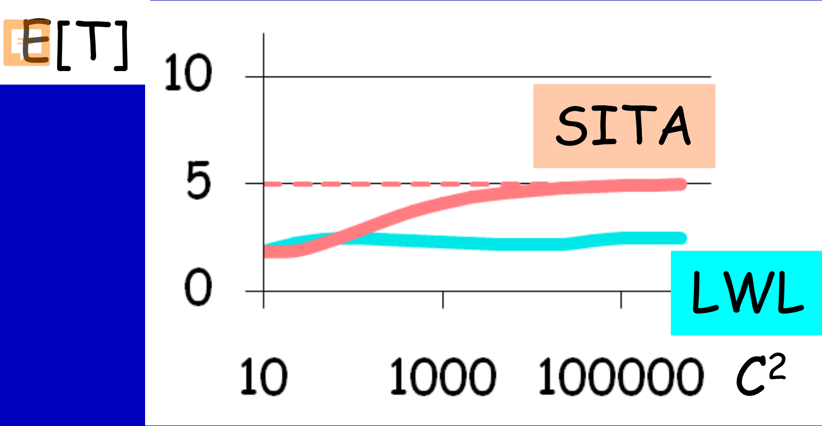
	Conv. LWL	Diverg. LWL
Conv. SITA	✓✓	✓✓
Diverg. SITA	✓✓	✓✓

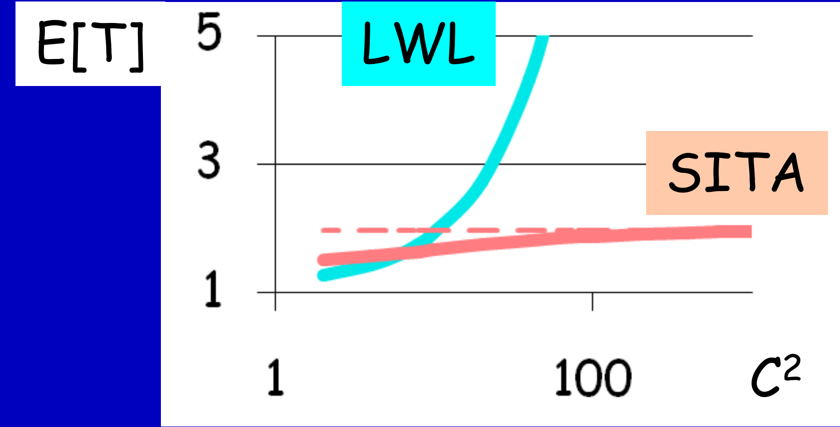




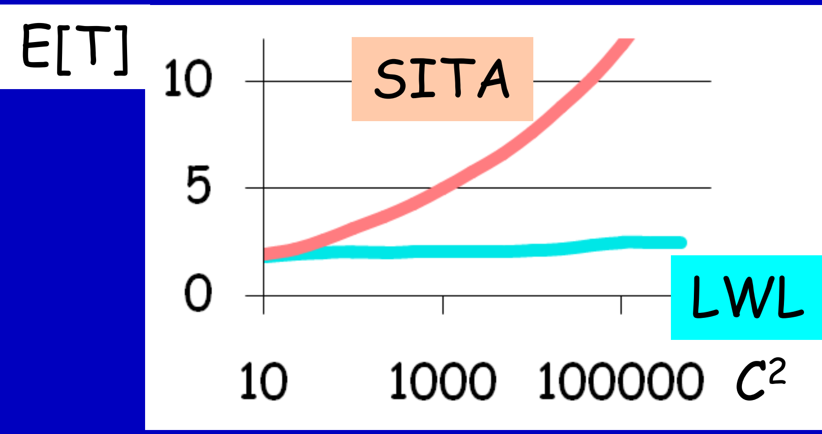
Way more complex, because job types overlap!

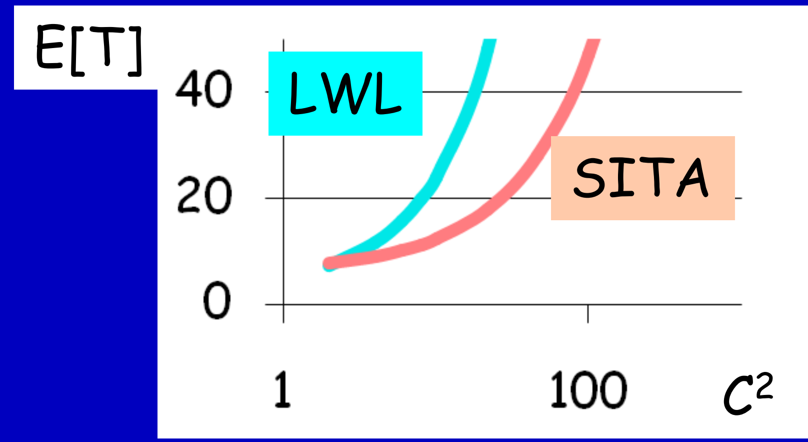
"Separation in the limit"





	Conv. LWL	Diverg. LWL
Conv. SITA	✓	✓
Diverg. SITA	✓	✓

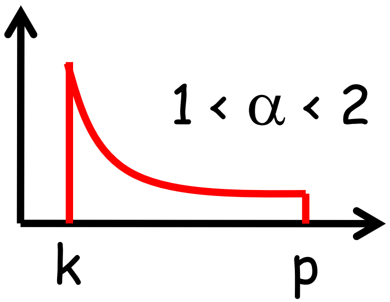




Bounded Pareto (2 server system)

	Conv. LWL	Diverg LWL
Conv. SITA		
Diverg SITA	on α	

$X \sim$ Bounded Pareto (k, p, α)



<u>Lemma</u>: As $C^2 \to \infty$, but $E[X]$, α: const,

$k \to (\alpha - 1)/\alpha \cdot E[X]$

$p \to \infty$

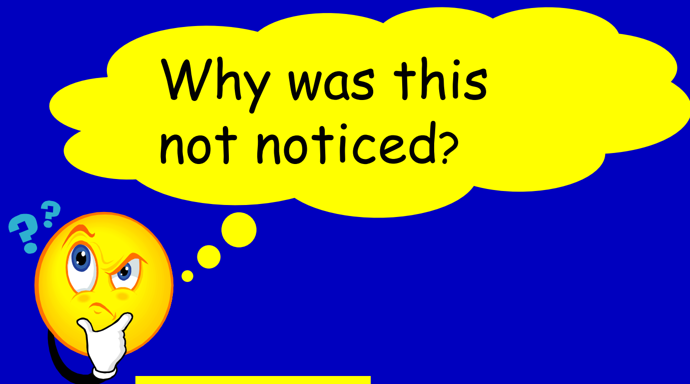
<u>THM</u>: SITA always diverges.

<u>THM</u>: If $\alpha > 3/2$ and $\rho < 1$, then LWL converges. Else LWL diverges.

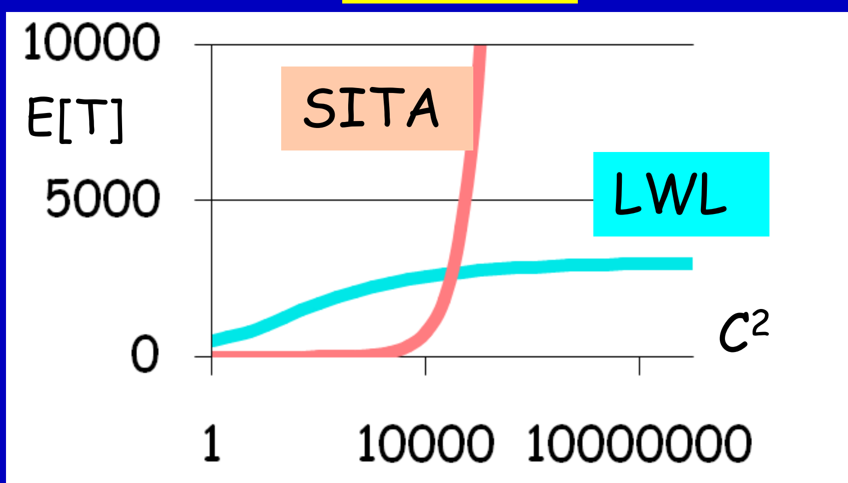
Extends to $n>2$ servers when $\rho < n-1$

Bounded Pareto Results

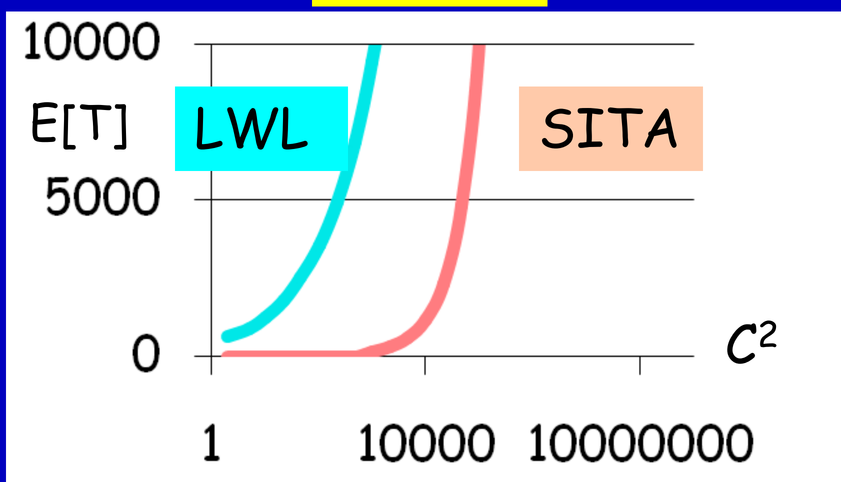
	Conv. LWL	Diverg. LWL
Conv. SITA		
Diverg. SITA	✓	✓



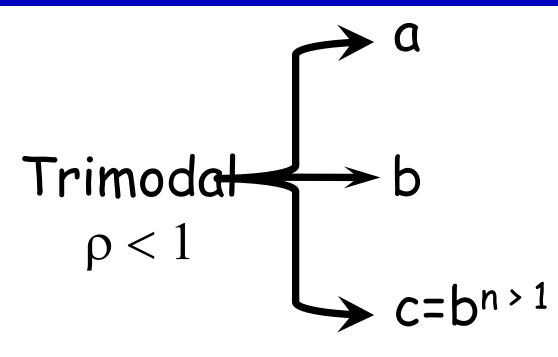
$\alpha = 1.6$



$\alpha = 1.4$



Summary



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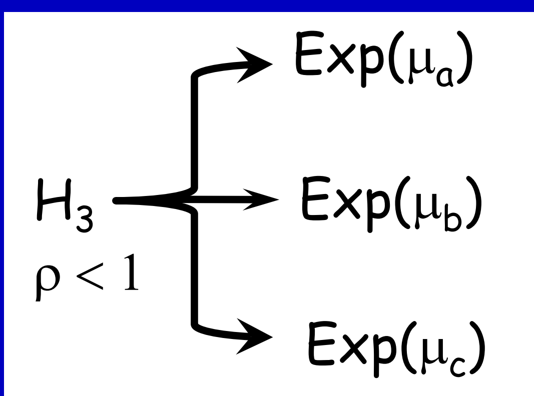
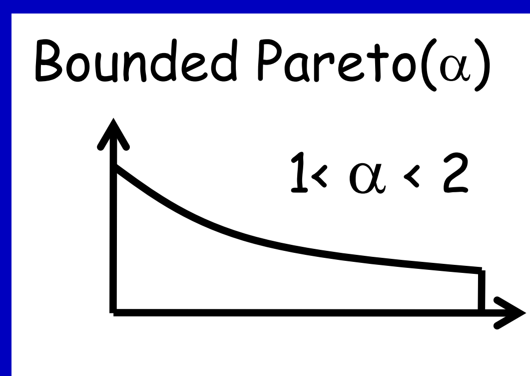
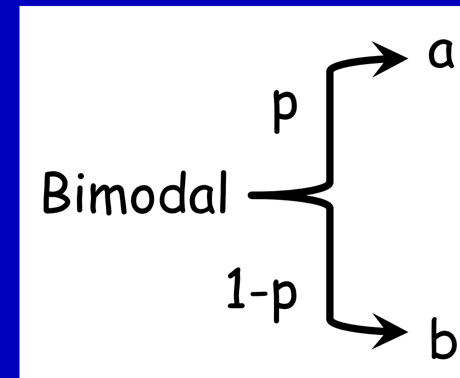


	Conv. LWL	Diverg. LWL
Conv. SITA	✓✓	✓✓
Diverg. SITA	✓✓✓	✓✓✓





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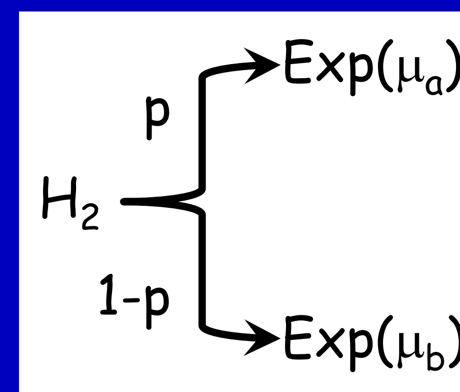

Old Nursery Rhyme

	Conv. LWL	Diverg. LWL
Conv. SITA	✓	✓
Diverg. SITA	✓	✓

When SITA is good, it is very, very good

But when it is bad, it is horrid.